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# Tiered neighborhood-exchange differential evolution for budget-constrained multi-root localization of nonlinear equation systems

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***Abstract.*** Budget-constrained localization of multiple roots of nonlinear equation systems requires both broad coverage of different attraction basins and rapid refinement of promising candidates when the number of residual evaluations is limited. Many niching variants of differential evolution perform replacement within local neighborhoods, but overly local mating can reduce basin coverage and cause premature stagnation. This paper introduces Tiered Neighborhood-Exchange Differential Evolution, a crowding-based solver that preserves neighborhood replacement while injecting controlled global information. The method uses a residual-gated dual mutation that switches between neighborhood exploitation and a global anchor, and a tiered neighborhood-exchange crossover that couples individuals across three fitness strata to counteract diversity loss. An archive of verified roots and distance-based duplicate filtering are employed to maintain a set of distinct solutions. Experiments on six benchmark systems show that, under identical evaluation budgets, the proposed method improves the recovered-root proportion and the probability of finding all distinct roots compared with representative niching differential-evolution baselines.

***Keywords:*** differential evolution, nonlinear equation systems, multi-root localization, niching, neighborhood exchange, evaluation budget, evolutionary computation.

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# Дифференциальная эволюция с многоуровневым обменом в окрестности для бюджетно-ограниченной локализации множества корней нелинейных систем уравнений

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***Резюме.*** Бюджетно-ограниченная локализация множества корней нелинейных систем уравнений требует одновременно охватывать различные области притяжения и быстро уточнять перспективные кандидаты при ограниченном числе вычислений невязки. Во многих нишевых вариантах дифференциальной эволюции замена выполняется внутри локальных окрестностей, однако чрезмерно локальное скрещивание снижает покрытие пространства и приводит к преждевременной стагнации. В работе предлагается дифференциальная эволюция с многоуровневым обменом в окрестности, которая сохраняет механизм замещения в окрестности, но вводит контролируемый обмен глобальной информацией. Метод использует мутацию с переключением по величине невязки, выбирая между локальной эксплуатацией и глобальным якорем, а также многоуровневое скрещивание, связывающее особей из трех фитнес-стратифицированных групп для поддержания разнообразия. Для формирования множества различных решений применяется архив подтвержденных корней и фильтрация дубликатов по расстоянию. Эксперименты на шести эталонных системах показывают, что предложенный подход при одинаковом вычислительном бюджете повышает долю обнаруженных корней и вероятность успешного нахождения всех корней по сравнению с репрезентативными нишевыми вариантами дифференциальной эволюции.

***Ключевые слова:*** дифференциальная эволюция, нелинейные системы уравнений, локализация множества корней, ниширование, обмен в окрестности, вычислительный бюджет, эволюционные вычисления.

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## Introduction

Nonlinear equation systems (NESs) appear in data mining, mechanical design, inverse modeling, and pattern recognition, where practitioners often care less about *a* feasible root than about *a collection* of distinct roots. A single operating point can be misleading; multiple roots may encode alternative designs, competing equilibria, or qualitatively different regimes. That practical requirement turns root finding into a budgeted, multimodal search problem.

Classical solvers – Newton-type methods, quasi-Newton variants, and homotopy continuation – remain valuable when a good initial guess is available and a single root is acceptable [1, 2]. In multi-root settings, however, these methods typically exhibit strong dependence on initialization and often produce one root per run; repeating them with different starts is possible, but the book-keeping becomes fragile as dimension grows.

Population-based metaheuristics offer a different trade-off. A single run can explore many basins concurrently, and parallelism is natural. Differential evolution (DE) is an especially common choice because its operators are simple, its search steps are easy to scale, and decades of parameter-control research exist [3]. Still, plain DE collapses diversity under selection pressure, so multi-root NES solvers almost always add explicit mechanisms: repulsion [4, 5], decomposition and reinitialization [6], speciation and clustering [7], and modern multitasking or transfer designs that share knowledge across subproblems [8, 9]. These directions have advanced the state of the art, yet a persistent practical issue remains: *coverage drift*. A run may begin with several niches, then lose some of them as the search becomes too local, or as neighborhoods become too insular.

Neighborhood crowding differential evolution (NCDE) is a useful baseline for multimodal search [10]. Its neighborhood-driven learning and crowding replacement can stabilize niches, but the same neighborhood emphasis can also trap individuals in shallow basins and reduce the chance of anchoring far-away roots when budgets are tight. The result is familiar: incomplete coverage, occasional root loss, and early stagnation.

In response, we propose *TNE-DE* (Tiered Neighborhood-Exchange Differential Evolution), a crowding-based multi-root solver that combines tiered neighborhood exchange with residual-gated dual mutation under a strict evaluation budget, while keeping the NCDE-style replacement backbone intact. The contributions are:

– A residual-gated dual-mutation operator that blends neighborhood exploitation with a controlled global injection to widen basin coverage while retaining a refinement-oriented perturbation near feasible regions.

– A tiered neighborhood-exchange crossover that assigns distinct inheritance partners to different fitness strata, deliberately importing outside-neighborhood coordinates for the weakest tier to refresh basin coverage.

– An NCDE-style crowding replacement with a verified-root archive for budgeted multi-root localization on NES benchmarks, with statistical comparison following standard nonparametric practice [11, 12].

## Materials and methods

*Problem formulation of nonlinear equation systems.* Consider an NES with $n$ variables and $m$ equations:

$$F(x) = 0, \quad F(x) \triangleq [f_1(x), f_2(x), \dots, f_m(x)]^\top, \tag{1}$$

where $x = (x_1, \dots, x_n) \in S$ and the bounded search space is

$$S = \prod_{i=1}^{n}[L_i, U_i] \subset \mathbb{R}^n. \tag{2}$$

A common evolutionary formulation converts root finding into residual minimization [4, 13]:

$$\min_{x \in S} \Gamma(x) \triangleq \| F(x) \|_2^2 = \sum_{j=1}^{m}(f_j(x))^2. \tag{3}$$

A candidate $x$ is accepted as a root when $\Gamma(x) \le \theta$, where $\theta > 0$ is a prescribed tolerance. Distinct roots are identified by filtering near-duplicates under a distance threshold (details follow the experimental protocol in Section "Experiments and Results").

In practice, distinct roots are identified using a distance-based filtering strategy. After all feasible solutions satisfying $\Gamma(x) \le \theta$ are collected, they are sorted in ascending order of residual. A greedy selection procedure is then applied: each candidate is compared against the accepted set, and it is retained only if its Euclidean distance to all previously accepted roots exceeds $\epsilon$. This ensures stable root counting and prevents duplicate detections.

*TNE-DE: Tiered neighborhood-exchange differential evolution.* Budgeted multi-root localization asks for two things at once: (i) persistent diversity across basins and (ii) careful use of evaluations so that a run does not spend most of its budget polishing a single niche. TNE-DE keeps the crowding replacement backbone of NCDE [10] but rewires the generation of trial candidates using (a) a dual-mutation rule driven by residual feedback and (b) tiered neighborhood-exchange crossover.

*Neighborhood construction.* At generation $g$, each individual $x_i(g)$ forms a neighborhood subpopulation $subpop_i(g)$ by selecting the $M - 1$ nearest individuals in Euclidean distance from the current population $pop(g)$. The neighborhood size decays with the evaluation counter to encourage wide sampling early and tighter local refinement late:

$$M(FES) = \left\lfloor 10 - 5 \cdot \frac{FES}{Max\_FES} \right\rfloor, \tag{4}$$

where $FES$ counts consumed function evaluations and $Max\_FES$ is the budget. This schedule keeps $M \in \{5, \dots, 10\}$, matching typical neighborhood scales used in crowding-based multimodal DE [10, 14].

*Residual-gated dual mutation.* Let $\Gamma_i = \Gamma(x_i(g))$ be the residual of $x_i(g)$ and let $\delta$ be a residual gate. Following common NES settings, we set $\delta = 0.5$ unless stated otherwise [4, 13]. The mutant vector $v_i(g + 1)$ is generated by one of two forms.

*Mode A: residual-large (coverage-biased).* When $\Gamma_i \ge \delta$, the mutation step borrows a global anchor while retaining neighborhood directionality:

$$v_i(g + 1) = x_{r_1}(g) + F(x_{r_2}(g) - x_{r_3}(g)), \tag{5}$$

where $x_{r_1}(g) \in pop(g)$ and $x_{r_2}(g), x_{r_3}(g) \in subpop_i(g)$, with mutually distinct indices and $r_k \neq i$. The scaling factor $F \in (0,2]$.

*Mode B: residual-small (basin refinement with "wake-up" perturbation).* When $\Gamma_i < \delta$, mutation targets refinement while leaving a controlled escape hatch:

$$v_i(g+1) = x_i(g) + F(x_{r_1}(g) - x_{r_2}(g)) + \Gamma_i(x_{r_3}(g) - x_{r_4}(g)), \tag{6}$$

where $x_{r_1}(g), x_{r_2}(g) \in subpop_i(g)$ and $x_{r_3}(g), x_{r_4}(g) \in pop(g)$, again with distinct indices. The last term is the key: near a strong basin, $\Gamma_i$ is small and the perturbation becomes negligible; near a weak or stagnant region, $\Gamma_i$ stays nontrivial and the global difference vector keeps injecting energy into the move.

In compact form,

$$v_i(g+1) = \begin{cases} x_{r_1}(g) + F(x_{r_2}(g) - x_{r_3}(g)), & \Gamma_i \geq \delta \\ x_i(g) + F(x_{r_1}(g) - x_{r_2}(g)) + \Gamma_i(x_{r_3}(g) - x_{r_4}(g)), & \Gamma_i < \delta \end{cases}. \tag{7}$$

The residual gate $\delta$ serves as a coarse feasibility indicator. Individuals with large residuals are encouraged to widen basin exploration through global anchoring (Mode A), while near-feasible individuals focus on local refinement with a residual-weighted perturbation term (Mode B). This design can be interpreted as an adaptive step-size modulation mechanism: the magnitude of the global perturbation automatically decreases as the solution approaches feasibility, while still preserving an escape component in weak or stagnating regions.

*Tiered neighborhood-exchange crossover.* TNE-DE assigns different inheritance partners to different fitness tiers. At each generation, the population is sorted by increasing residual (smaller is better) and split into three fixed-ratio groups: superior (SP), middle (MP), and inferior (IP). For individual $x_i(g)$, define a reference donor $z_i(g)$ as

$$z_i(g) = \begin{cases} x_{best}^{in}(g), & x_i(g) \in SP \text{ or } \Gamma_i < \delta \\ x_{rand}^{in}(g), & x_i(g) \in MP \\ x_{rand}^{out}(g), & x_i(g) \in IP \end{cases}, \tag{8}$$

where $x_{best}^{in}(g)$ is the best individual in $subpop_i(g)$, $x_{rand}^{in}(g) \in subpop_i(g) \backslash \{x_i(g)\}$ is a random neighbor, and $x_{rand}^{out}(g) \in pop(g) \backslash subpop_i(g)$ is a random individual outside the neighborhood. The IP case is deliberately "nonlocal": it injects coordinates from outside the neighborhood to refresh basin coverage.

Given $v_i(g+1)$ and $z_i(g)$, TNE-DE uses binomial crossover to form the trial $u_i(g+1)$:

$$u_{i,j}(g+1) = \begin{cases} v_{i,j}(g+1), & \text{rand}(0,1) \leq CR \text{ or } j = j_{rand} \\ z_{i,j}(g), & \text{otherwise} \end{cases}, \tag{9}$$

where $CR \in [0,1]$ and $j_{rand}$ forces at least one coordinate from the mutant.

*Overall procedure.* TNE-DE integrates (7)–(9) into an NCDE-style crowding replacement. A root archive $X_{save}$ stores accepted roots; whenever $\Gamma(u_i) \leq \theta$, the found root is saved and the replaced slot is reinitialized to keep the run from burning the remaining budget on a single basin (Algorithm 1).

Algorithm 1 – TNE-DE for Nonlinear Equation Systems
Алгоритм 1 – Дифференциальная эволюция с многоуровневым обменом в окрестности для нелинейных систем уравнений

**Input:** population size $NP$, evaluation budget $Max\_FES$, space $S$, dimension $n$, tolerance $\theta$
**Output:** a set of distinct roots $x^*$
Initialize $pop(0) = \{x_i(0)\}_{i=1}^{NP}$ uniformly at random in $S$
Evaluate $\Gamma\big(x_i(0)\big)$ for all $i$; set $FES \leftarrow NP$
Set generation counter $g \leftarrow 0$
Initialize archive $X_{save} \leftarrow \emptyset$
While $FES < Max\_FES$
Sort $pop(g)$ by ascending $\Gamma$; split into SP, MP, IP by preset ratios
For $i = 1$ to $NP$
Build neighborhood $subpop_i(g)$ using (4)
Generate mutant $v_i(g+1)$ via (7)
Select donor $z_i(g)$ via (8)
Generate trial $u_i(g+1)$ via (9)
Evaluate $\Gamma\big(u_i(g+1)\big)$; $FES \leftarrow FES + 1$
Find $x_near(g)$, the individual in $pop(g)$ with minimum Euclidean distance to $u_i(g+1)$
Replace $x_{near}(g)$ by $u_i(g+1)$ if $\Gamma\big(u_i(g+1)\big) \le \Gamma\big(x_{near}(g)\big)$ (crowding selection)
If $\Gamma\big(u_i(g+1)\big) \le \theta$
Store $u_i(g+1)$ into $X_{save}$; reinitialize the replaced individual uniformly in $S$
End if
End for

$$g \leftarrow g + 1$$

End while
Merge feasible roots in $pop(g)$ with $X_{save}$; filter near-duplicates; return final root set $x^*$

*Theoretical Analysis*

*1) Time complexity.* The dominant overhead comes from (i) distance computations for neighborhood construction and crowding replacement and (ii) sorting by residual. Ignoring lower-order terms, the time complexity is

$$O(T \cdot NP^2 \cdot \max\{n, \log NP\}),$$

which matches NCDE under the same neighborhood-and-crowding structure [10].

*2) Multi-root localization behavior.* Dual mutation injects global information in two distinct ways: a global anchor in Mode A and a residual-weighted perturbation in Mode B. Tiered neighborhood-exchange crossover then controls how that information is inherited: SP tightens basins, MP keeps mild variability, and IP imports outside-neighborhood coordinates to prevent coverage drift. In practice, these pieces reduce root loss while keeping enough global motion to claim late-discovered basins, a pattern reported as critical in modern NES solvers [9, 13]. Notably, Wang et al. developed a hybrid niching-based differential evolution with two archives for nonlinear equation systems, highlighting the practical importance of maintaining multiple basins during the search [15].

*Experimental setup. Benchmark functions.* To evaluate multi-root localization, six NES benchmarks with diverse characteristics are considered:

$$F_1\colon \begin{cases} x_1^2 - x_2^2 = 0 \\ 1 - |x_1 - x_2| = 0 \end{cases}, \ (x_i \in [-3{,}3], \text{2 roots}),$$

$$F_2: \begin{cases} x_1 - \cos(4\pi x_2) = 0 \\ x_1^2 + x_2^2 - 1 = 0 \end{cases}, \ (x_i \in [-1,1], \text{15 roots}),$$

$$F_3: \begin{cases} 0.04\left(\frac{11}{15} - x_1\right) e^{\frac{16x_1}{1+x_1/0.04}} - x_1 = 0 \\ 0.04(2.2 - 2x_1 - 3x_2) e^{\frac{16x_1}{1+x_2/0.04}} + x_1 - 3x_2 = 0 \end{cases}, \ (x_i \in [0,1], \text{7 roots}),$$

$$F_4: x_i - \cos\left(2x_i - \sum_{j=1}^{n} x_j\right) = 0,\ i = 1,2,\dots,n,\ (x_i \in [-1,1], n = 3, \text{7 roots}),$$

$$F_5: \begin{cases} 4x_1^3 - 3x_1 - x_2 = 0 \\ x_1^2 - x_2 = 0 \end{cases}, \ (x_1 \in [-5,1.5], x_2 \in [0,5], \text{3 roots}),$$

$$F_6: \begin{cases} x_1^2 - 2x_1 - x_2^2 + x_3^2 = 0 \\ \sin\left(x_2 - \exp(x_1)\right) = 0 \\ x_3 - \log|x_2| = 0 \end{cases}, \ (x_1 \in [0,2], x_2 \in [-10,10], x_3 \in [-1,1], \text{5 roots}).$$

*Metrics and parameter settings.* Root rate (RR) and success rate (SR) are adopted to evaluate multi-root localization performance. Let $K$ denote the total number of known distinct roots for a benchmark problem, and let $\widehat{K}$ denote the number of distinct roots found in a single run under tolerance $\theta$. The root rate is defined as

$$RR = \widehat{K}/K, \tag{10}$$

which measures the proportion of successfully recovered roots. The success rate (SR) is defined as the fraction of independent runs (out of 30) in which all roots are located, i.e., $\widehat{K} = K$.

The compared algorithms include NCDE [10], KSDE [7], CADE [16], FNODE [17], ANDE [18], and self-CCDE [14]. All algorithms use population size $NP = 100$ and are independently executed 30 times on each benchmark. The root tolerance is set to $\theta = 10^{-6}$. Two solutions are considered identical if their Euclidean distance is smaller than $\epsilon$; duplicate solutions are removed using a greedy filtering procedure that retains the lower-residual candidate.

The residual gate is fixed to $\delta = 0.5$. On the considered benchmarks, the squared residual values are bounded and typically fall within a comparable numerical scale due to normalized variable ranges. Therefore, a fixed residual threshold provides a consistent criterion to distinguish far-from-feasible individuals from near-feasible ones. Adaptive gate control remains an interesting direction for future work.

For DE control parameters, we use $F = 0.5$ and $CR = 0.9$ for methods employing standard DE operators unless otherwise specified in the original papers. The default tier ratio for TNE-DE is SP:MP:IP = 3:1:1, selected according to the ratio study.

For statistical comparison, we first apply the Friedman test at significance level $\alpha = 0.05$. When the null hypothesis is rejected, pairwise Wilcoxon signed-rank tests with Holm correction are conducted between TNE-DE and each baseline, and corrected p-values are used to determine statistical significance.

## Results and discussion

*1) Impact of grouping ratios.* To study the effect of SP:MP:IP ratios, ten configurations are tested (e.g., 1:1:1, 1:1:2, etc.). The Friedman ranking indicates that TNE-DE-10 (ratio 3:1:1) provides the best overall ordering on both RR and SR (Table 1).

Table 1 – Friedman rankings of TNE-DE under different grouping ratios
Таблица 1 – Ранжирование по критерию Фридмана для алгоритма дифференциальной эволюции с многоуровневым обменом в окрестности при различных соотношениях групп

| Algorithm | Ranking (RR) | Ranking (SR) |
|---|---|---|
| TNE-DE-1 (1:1:1) | 4.0000 | 4.1667 |
| TNE-DE-2 | 2.6667 | 2.5000 |
| ... | ... | ... |
| TNE-DE-10 (3:1:1) | **1.6667** | **1.6667** |

*2) Ablation study on the proposed strategies.* We compare TNE-DE-DM (dual mutation only), TNE-DE-TX (tiered neighborhood-exchange crossover only), and the full TNE-DE. The integrated method yields consistently higher RR than NCDE, supporting the complementarity of mutation gating and tiered exchange (Table 2).

Table 2 – Comparison results in terms of RR
Таблица 2 – Результаты сравнения по показателю доли обнаруженных корней

| Function | NCDE | TNE-DE-DM | TNE-DE-TX | TNE-DE |
|---|---|---|---|---|
| $F_1$ | 0.4500 | 0.4500 | 0.7167 | **0.7667** |
| $F_2$ | 0.8844 | 0.9600 | 0.9511 | **0.9822** |
| $F_3$ | 0.8762 | 0.9952 | 0.9619 | **1.0000** |
| ... | ... | ... | ... | ... |
| Avg. | 0.8358 | 0.9582 | ... | **0.9582** |

*3) Overall comparison with competing methods.* Across the six benchmarks, TNE-DE achieves the highest RR values and obtains an average RR of 0.9582. Friedman rankings confirm that TNE-DE attains the top overall rank among all compared methods (Table 3).

Table 3 – Friedman rankings of all algorithms
Таблица 3 – Ранжирование по критерию Фридмана для всех алгоритмов

| Algorithm | Ranking (RR) | Ranking (SR) |
|---|---|---|
| **TNE-DE** | **1.0000** | **1.0000** |
| NCDE | 3.6667 | 4.0000 |
| KSDE | 2.5000 | 2.5000 |
| ANDE | 6.0000 | 5.8333 |
| FNODE | 3.8333 | 4.0000 |
| CADE | 2.6667 | 2.8333 |
| self-CCDE | 3.6667 | 3.8333 |

## Conclusion

This paper proposed TNE-DE, a tiered neighborhood-exchange DE solver with residual-gated dual mutation for budgeted multi-root solving of nonlinear equation systems. The method keeps the crowding replacement backbone of NCDE while changing how candidates are generated: mutation switches its information sources based on residual feedback, and crossover assigns nonidentical inheritance partners to different fitness tiers, including explicit outside-neighborhood exchange for weak individuals. Experiments on six NES benchmarks show improved root coverage and success frequency compared with representative niching DE baselines.

Future work will study adaptive ratio control for the tier split, as well as repulsion-and-aggregation hybrids that can scale to harder high-dimensional NES instances reported in recent test suites [9, 19].

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